

Diversity and Degrees of Freedom of Cooperative Wireless Networks

K. Sreeram

Department of ECE
Indian Institute of Science
Bangalore, India

Email: sreeramkannan@ece.iisc.ernet.in

S. Birenjith

Department of ECE
Indian Institute of Science
Bangalore, India

Email: biren@ece.iisc.ernet.in

P. Vijay Kumar

Department of ECE
Indian Institute of Science
(on leave from USC)

Email: vijayk@usc.edu

Abstract—Wireless fading networks with multiple antennas are typically studied information-theoretically from two different perspectives - the outage characterization and the ergodic capacity characterization. A key parameter in the outage characterization of a network is the diversity, whereas a first-order indicator for the ergodic capacity is the degrees of freedom (DOF), which is the pre-log coefficient in the capacity expression. In this paper, we present max-flow min-cut type theorems for computing both the diversity and the degrees of freedom of arbitrary single-source single-sink multi-antenna networks. We also show that an amplify-and-forward protocol is sufficient to achieve this. The degrees of freedom characterization is obtained using a conversion to a deterministic wireless network for which the capacity was recently found. We show that the diversity result easily extends to multi-source multi-sink networks and evaluate the DOF for multi-casting in single-source multi-sink networks.

I. INTRODUCTION

There has been a recent interest in determining the degrees of freedom (DOF) of wireless multi-antenna networks [3]. For definitions of diversity and degrees of freedom of point-to-point channels, see [6] for example. The DOF for the N user interference channel was recently derived in [4] and the DOF of single-source single-sink layered networks was obtained in [7].

We characterize the diversity for arbitrary networks and compute degrees of freedom (DOF) for single-source single-sink and multi-cast networks with multiple antennas. We compute the degrees of freedom using a connection to deterministic wireless networks. The capacity of single-source single-sink and multi-cast deterministic wireless networks were characterized in [1]. Intuition drawn from the deterministic wireless networks were used to identify capacity to within a constant for some example networks in [2]. A similar approach was used in [5] for obtaining DOF for real gaussian interference networks.

While the results for wire-line finite-field single-source single-sink network have been known for some time [9], multi-cast capacity was found in the more recent seminal work [10]. An algebraic approach for finding the multi-cast capacity was given in [11]. These results were extended to finite field wireless networks in [1]. In [12], computation codes were used to study the capacity of finite field networks with interference alone. While it is easy to extend wireline finite field network

TABLE I
NETWORK CODING FOR FINITE FIELD AND GAUSSIAN NETWORKS

	Wireline Networks		Wireless Networks	
	Capacity of Finite Field Networks	DOF of Gaussian Networks	Capacity of Finite Field Networks	DOF of Gaussian Networks
Single Source	Min-cut [9]	Min-cut (easy to see)	Min-cut rank [1]	Min-cut rank (this paper)
Multicast	Minimum min-cut [10]	Minimum min-cut (easy to see)	Minimum min-cut rank [1]	Minimum min-cut rank (this paper)

results to gaussian wireline networks, the extension of wireless finite field network results to the gaussian case is not obvious. In this paper, we apply these finite field network results to compute the DOF and diversity for gaussian wireless networks. Table I summarizes these developments.

The diversity of a family of multi-hop networks was evaluated in [14]. In [13], the diversity for two-hop MIMO relay channel with a certain condition on the number of antennas. However the maximum diversity of an arbitrary network remains an open question, which we settle in this paper.

A. Representation of a Network

We represent a single-antenna wireless network by a edge labelled directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where \mathcal{V} is the set of vertices and \mathcal{E} is the set of edges. Each node is represented by a vertex, each edge represents a transmission link. Let $N := |\mathcal{E}|$ be the number of links in the network. The label on every edge $\mathcal{L}(e)$, $e \in \mathcal{E}$ represents the fading coefficient on that transmission link. By convention, we put an edge only when the link has non-zero fading coefficient.

In the case of multiple antenna networks, we first pass on to an equivalent representation, where every terminal is represented by a super-node and every antenna attached to the terminal is represented by a small node associated with the super-node. Edges representing single-antenna connections are drawn only between small nodes and hence we can still

label edges by scalar fading coefficients. For the gaussian network, we assume that coefficients are elements of the complex field \mathbb{C} . Since we are dealing with wireless networks, we assume that the broadcast and interference constraints hold. We assume throughout the paper that CSIR is present. We also assume for the degrees of freedom part, that CSIT is present.

Definition 1: A cut ω between source S_i and destination D_j on a multiple-antenna gaussian network is defined as a partition of the super-nodes into U and U^c such that the source S_i is present in U and D_j is present in U^c . Let the set of all cuts between source S_i and destination D_j be denoted by Ω_{ij} . Given a cut ω , the matrix of the cut, H_ω is defined as the transfer matrix associated with edges crossing the cut from the source side to destination side. In the single source single sink case, we will drop the unneeded ij suffix.

Remark 1: Any wire-line network can be converted into a wireless network, by adding a sufficient number of small-nodes at each super-node thereby separating the links so that in effect, the broadcast and interference constraints are nullified. We call this the natural embedding of a wire-line network into a wireless network.

B. Degrees of Freedom

Definition 2: Consider a single-sink wireless network with each node having a symmetric transmit power constraint, ρ . Let the capacity between the source i and the sink j be $C_{ij}(\rho)$. The *degrees of freedom* of the flow between source i and sink j is defined as

$$D_{ij} = \lim_{\rho \rightarrow \infty} \frac{C_{ij}(\rho)}{\log \rho} \quad (1)$$

Remark 2: The capacity of the flow F_{ij} between can source i and sink j can then be given by:

$$C_{ij} = D_{ij} \log(\rho) + o(\log \rho) \quad (2)$$

Whenever we consider a single-source single-sink network, we will without loss of ambiguity, drop the suffix from C_{ij} , D_{ij} and simply use C , D instead.

Definition 3: A *multi-cast* network is defined as a network with single-source and multiple-sinks, with the constraint that all the flows need the same information from the source.

Lemma 1.1: Consider a channel of the form $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{W}$, where H is a $N \times N$ channel matrix X, Y, W are N length column vectors representing the transmitted signal, received signal and the noise vector distributed as $\mathcal{CN}(0, \Sigma)$, where Σ is a full rank correlation matrix. The degrees of freedom of this channel is given by $D = \text{rank}(H)$.

Remark 3: We define the DOF of a matrix H as the DOF of the channel $Y = HX + W$ where W is $\mathcal{CN}(0, I)$.

C. Diversity

The Lemma below computes the diversity of a channel matrix having a specific structure.

Lemma 1.2: Consider a channel of the form $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{W}$, where H is a $N \times N$ random channel

matrix, X, Y, W are N length column vectors representing the transmitted signal, received signal and the noise vector. Let the noise vector W can be representable as $W = Z + Z_0 + \sum_{i=1}^L G_i Z_i$, where Z_i are $\mathcal{CN}(0, I)$ independent vectors and every entry in the matrices G_i are polynomial functions of gaussian random variables. If the N^2 entries of matrix H contain exactly M independent Rayleigh fading coefficients, then M is the diversity of that matrix.

D. Linear Deterministic Wireless Network

In defining deterministic¹ wireless networks, we follow [1]. These networks are similar to multiple-antenna gaussian networks with the only difference being that these networks are noise-free and that the complex fading coefficients are replaced by finite fields elements. In place of complex vectors, each node transmits an q -tuple over the finite field. Cuts are defined as in the gaussian network case. In place of H_ω , we use G_ω to define the transfer matrix between nodes on either side of the cut ω . We state the following Theorem from [1]:

Theorem 1.3: [1] Given a linear deterministic single-source single-sink wireless network over any finite field \mathbb{F} , $\forall \epsilon > 0$, the ϵ -error capacity C of such a relay network is given by,

$$C = \min_{\omega \in \Omega} \text{rank}(G_\omega). \quad (3)$$

where the capacity is specified in terms of the number of finite field symbols per unit time. A strategy utilizing only linear transformations over \mathbb{F} at the relays is sufficient to achieve this capacity.

Remark 4: The strategy specified in [1] utilizes matrix transformations at each relay of the input vector received over a period of T time slots. Thus the achievability shows the existence of relay matrices A_i at each relay node $i \in \mathcal{V}$, each of size $qT \times qT$, that specifies the transformation between the received vector of size qT to the vector of size qT that needs to be transmitted. It can be seen using the natural embedding of a wire-line network into a wireless network, that this theorem is indeed a generalization of the max-flow min-cut theorem. The multicast-version of Theorem 1.3 appears below.

Theorem 1.4: [1] Given a linear deterministic single-source D - sink multi-cast wireless network, $\forall \epsilon > 0$, the ϵ -error capacity C of such a network is given by,

$$C = \min_{j=1,2,\dots,D} \min_{\omega \in \Omega_j} \text{rank}(G_\omega). \quad (4)$$

where Ω is the set of all cuts between the source and destination j . A strategy utilizing only linear transformations at the relays is sufficient to achieve this capacity.

II. MIN-CUT EQUALS MAX DIVERSITY

We begin with a result applicable to gaussian networks.

Definition 4: We define the value M_ω of a cut ω as the number of edges crossing over from the source side to the sink side across the cut. We refer to the value of the min-cut as simply the min-cut.

¹By deterministic network, we will always mean linear deterministic network.

Theorem 2.1: Consider a multi-terminal fading network with nodes having multiple antennas with each edge having iid Rayleigh-fading coefficients. The maximum diversity achievable for any flow is equal to the min-cut between the source and the sink corresponding to the flow. Each flow can achieve its maximum diversity simultaneously.

Proof: We will distinguish between two cases.

Case I: Network with single antenna nodes

Choose a source S_i and sink D_j . Let \mathcal{C}_{ij} denote the set of all cuts between S_i and D_j .

From the cutset bound [15] on DMT [16],

$$\begin{aligned} d(0) &\leq \min_{\omega \in \Omega_{ij}} \{d_\omega(0)\} = \min_{\omega \in \Omega_{ij}} \{M_\omega\} \\ \Rightarrow d(0) &\leq M \end{aligned}$$

It is now sufficient to prove that diversity order of M is achievable. Let us first consider the case when there is only one flow.

By the Ford-Fulkerson theorem [9], the number of edges in the min-cut is equal to the maximum number of edge disjoint paths between source and the destination. Schedule the network in such a way that each edge in a given edge disjoint path is activated one by one. Repeat for all the edge disjoint paths. Thus the same data symbol is transmitted through all the edge disjoint paths from S_i to D_j . Let the number of edges in the i -th edge disjoint path be n_i . The j th edge in the i th edge disjoint path is denoted by e_{ij} and the associated fading coefficient be h_{ij} . The activation schedule can be represented as follows: Activate each of the following edge individually in successive time instants: $e_{11}, e_{12}, \dots, e_{1n_1}, e_{21}, \dots, e_{2n_2}, \dots, e_{M1}, e_{M2}, \dots, e_{Mn_M}$. Now define $h_i := \prod_{j=1}^{n_i} h_{ij}$ to be the product fading coefficient on the i -th path. Let the total number of time slots required be $N = \sum_{i=1}^M n_i$.

With this protocol in place, the equivalent channel seen from the source to the destination has channel matrix

$$H = \begin{bmatrix} h_1 & 0 & \dots & 0 \\ 0 & h_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & h_M \end{bmatrix}$$

This matrix is exactly of the structure in Lemma 1.2 except that there are M product Rayleigh coefficients. However, it can be shown that the diversity remains unchanged. The noise matrix also obeys the conditions of Lemma 1.2. Thus the maximum diversity of M can be achieved.

When there are multiple flows in the network, we simply schedule the data of all the flows in a time-division manner. This will entail further rate loss - however, since we are interested only in the diversity, we can still achieve each flow's maximum diversity simultaneously.

Case II: Network with multiples antenna nodes

In the multiple antenna case, we regard any link between a n_t transmit and n_r receive antenna as being composed of $n_t n_r$

links, with one link between each transmit and each receive antenna. Note that it is possible to selectively activate precisely one of the $n_t n_r$ Tx-antenna-Rx-antenna pairs by appropriately transmitting from just one antenna and listening at just one Rx antenna. The same strategy as in the single antenna case can then be applied to achieve this diversity in the network. ■

III. DEGREES OF FREEDOM OF SINGLE SOURCE SINGLE SINK NETWORKS

In this section, we present a max-flow min-cut type theorem for evaluating the degrees of freedom in single-source single-sink networks:

Theorem 3.1: Given a single-source single-sink gaussian wireless network, with independent fading coefficients having an arbitrary density function, the DOF of the network is given by

$$D = \min_{\omega \in \Omega} \text{rank}(H_\omega) \text{ with probability one.} \quad (5)$$

An amplify-and-forward strategy utilizing only linear transformations at the relays is sufficient to achieve this DOF.

Remark 5: If we assume that the channel coefficients have a time variation, then we can show that the degrees of freedom is identically equal to the value shown in the theorem, by coding across the time variation.

Proof: The proof proceeds as follows:

- 1) First, a converse for the DOF is provided using simple cutset bounds.
- 2) Then, we convert the gaussian network into a deterministic network with the property that the cutset bound on DOF for the gaussian network is the same as the cutset bound on the capacity of the deterministic network.
- 3) We then characterize the zero-error-capacity of the linear deterministic wireless network.
- 4) Finally, we convert the achievability result for the deterministic network into an achievability result for gaussian network, which matches the converse.

■

A. The Converse

We first provide a simple converse on the degrees of freedom of a single source single sink network.

Lemma 3.2: Given a single-source single-sink network, the DOF is upper bounded by the DOF of every cut:

$$D \leq \min_{\omega \in \Omega} \text{rank}(H_\omega)$$

where H_ω is the matrix corresponding to the cut ω .

B. Conversion to Linear Deterministic Network

In this subsection, we convert the wireless gaussian network to a equivalent linear deterministic network.² We use the term "equivalent" to signify that the DOF of the gaussian network and the capacity of the deterministic network are the same. In

²It must be noted that the conversion to deterministic network used here is different from that used in [2] and [5].

order to get the equivalent deterministic network, we proceed as follows:

Let the fading coefficients on the N edges in the gaussian network be h_1, h_2, \dots, h_N . We first consider a finite field network with the same graph as the original gaussian network. We take q , the vector length in the deterministic network to be equal to the maximum number of antennas of any node in the gaussian network. For nodes with antennas less than q , we leave the remaining nodes un-connected. However, we still need to decide the finite field size, p , and a finite field coefficient on each edge. Given a finite field size, we need N maps, $\psi_i, i = 1, 2, \dots, N$ that convert the gaussian fading coefficients into finite field coefficients. Let $\xi_i := \psi_i(h_i)$ denote this mapping.

In order to obtain these coefficients and the finite field size, we require further conditions. In particular, we will require the finite field network to have at least the same capacity as the upper bound on the gaussian network. We recognize the similarity between the capacity equation in Theorem 1.3 and DOF terms in Lemma 3.2 and require that cut by cut, the rank of the transfer matrix on deterministic network be no less than the rank on the gaussian network. Before assigning values to ξ_i , we will treat them as formal variables.

Consider a cut ω in the gaussian network. We want the $\text{rank}(G_\omega) \geq \text{rank}(H_\omega)$ for this cut. To do this, let $r_\omega := \text{rank}(H_\omega)$ be the DOF of the cut in the gaussian network. Then there exists a $r_\omega \times r_\omega$ sub-matrix of the H_ω which has non-zero determinant. Let us call this matrix as H'_ω . Consider the same cut on the deterministic network and find the same $r_\omega \times r_\omega$ sub-matrix G'_ω corresponding to the transfer matrix G_ω . Now consider the determinant of the matrix G'_ω . The determinant is a polynomial in several variables $\xi_i, i = 1, 2, \dots, N$ with rational integer coefficients. Let us call this polynomial as $f_\omega(\xi_1, \xi_2, \dots, \xi_N)$.

This polynomial is not identically zero as a polynomial over \mathbb{Q} , since in that case even the substitution $\xi_i = h_i$ will lead to a zero value, making the determinant zero even for the gaussian case, which is clearly a contradiction. Therefore we have that f_ω is a non-zero polynomial. We also have an observation that the degree of f_ω in each of the variable ξ_i is at-most one. We want a field \mathbb{F}_p and an assignment to ξ_i that makes the f_ω non-zero over the chosen field. For any given cut, this can be easily done. However we want to do this simultaneously for all cuts. To do so, we will employ the following lemma, proven easily using elementary algebra:

Lemma 3.3: Given a polynomial $f(\xi_1, \xi_2, \dots, \xi_N)$ with integer coefficients, which is not identically zero, there exists a prime field \mathbb{F}_p with p large enough, such that the polynomial evaluates to a non-zero value at least for one assignment of field values to the formal variables.

Now consider the polynomial

$$f(\xi_1, \xi_2, \dots, \xi_N) := \prod_{\omega \in \Omega} f_\omega(\xi_1, \xi_2, \dots, \xi_N) \quad (6)$$

Now, the polynomial f is non-zero since it is a product of non-zero polynomials f_ω and the degree of f in any of the

variables is at-most $|\Omega|$. We want a field \mathbb{F}_p and an assignment for ξ_i from the field such that f is non-zero. Using Lemma 3.3, we have that such an assignment exists. Let us choose that p and the assignment that makes f non-zero. Thus we have a deterministic wireless network whose capacity is guaranteed to be greater than or equal to γ of the converse.

C. Zero Error Capacity of Deterministic Networks

We establish the zero error capacity of deterministic wireless networks. We have the following definition

Definition 5: [8] The zero error capacity is defined as the supremum of all achievable rates such that the probability of error is exactly zero.

Theorem 3.4: The zero error capacity of a single source single sink deterministic wireless network is equal to

$$C_{ZE} = \min_{\omega \in \Omega} \text{rank}(G_\omega)$$

This capacity can be achieved using a linear code and linear transformations in all relays.

Proof: We will prove this theorem using the ϵ error capacity result from Theorem 1.3. We will assume the field \mathbb{F} appearing in the theorem to be the finite field \mathbb{F}_p of size p where p is the prime previously identified. From the achievability result in the proof of Theorem 1.3, we have that given any $\epsilon > 0$ and rate $r < C$, there exists a block-length T , linear transformations $A_j, j = 1, 2, \dots, M$ of size $qT \times qT$ used by all relays and a code book \mathcal{C} for the source, such that the probability of error is lesser than or equal to ϵ . Each codeword $X_i \in \mathcal{C}$ is a $qT \times 1$ vector that specifies the entire transmission from the source. Let $X_1, \dots, X_{|\mathcal{C}|}$ be the codewords.

Let us assume that the sink listens for a duration $T' \geq T$ in general to account for the presence of paths of unequal lengths in the network between source and sink, (for large T , we would have $\frac{T'}{T} \rightarrow 1$, so this does not affect rate calculations). The transfer equation between the source and the destination vectors are specified by: $Y = GX$ since all transformations in the network are indeed linear. Here G is a $qT' \times qT$ matrix, X is a T length transmitted vector, and Y is a T' length vector.

Now, given that a vector X_i is transmitted, either the decoder always makes an error or never makes error because the channel is a deterministic map $Y_i = GX_i$. Let P_e^i be the probability of error conditioned on transmitting the i -th codeword. Then $P_e^i \in \{0, 1\}$ and the average codeword error probability

$$P_e = \frac{1}{|\mathcal{C}|} \sum_{i=1}^{|\mathcal{C}|} P_e^i \leq \epsilon \Rightarrow \sum_{i=1}^{|\mathcal{C}|} P_e^i \leq \epsilon |\mathcal{C}|$$

This means that at least $(1 - \epsilon)|\mathcal{C}|$ codewords have zero probability of error. Therefore if we choose only these $(1 - \epsilon)|\mathcal{C}|$ codewords as an expurgated code-book \mathcal{C}' , then the code-book has zero probability of error under the same relay matrices and decoding rule. The rate of the codebook is however $\bar{r} = r - \frac{\log(1-\epsilon)}{T}$. Let $\delta = \frac{\log(1-\epsilon)}{T}$ be the rate loss and therefore, the expurgated code-book has negligible rate loss as T becomes large. Now, we have established a zero error

codebook of rate $r - \delta$. By choosing r arbitrarily close to C and T large, we get that indeed $C_{ZE} = C$.

However, the code \mathcal{C}' like the code \mathcal{C} used in [1], is a non-linear code. We obtain a linear code by utilizing the following technique: Since there is a zero error code for rate \bar{r} , it means that the transfer matrix G has rank at least $\bar{r}T$ and therefore that there is a sub-matrix G' of size $\bar{r}T \times \bar{r}T$, which is full rank. If we communicate only on these $\bar{r}T$ dimensions we can obtain the transfer matrix G' . Thus we get a linear zero error code of rate \bar{r} . ■

D. Achievable DOF in Gaussian Networks

In this sub-section, we will lift the zero-error-capacity achievability result from deterministic networks to determine an achievable DOF for gaussian networks.

In the achievability for capacity of deterministic networks, the relays performed matrix operations A_i on received vectors for T time durations. Since each received vector is of size q , the matrix A_i is of size $qT \times qT$. Now we use the same strategy for the gaussian network, i.e., all relays use the same matrices A_i that they used in the deterministic network. This makes sense, since in a prime finite field \mathbb{F}_p , all field elements are integers modulo p . Therefore the matrices A_i can also be interpreted as matrices over \mathbb{C} . This strategy yields a effective channel matrix H , i.e., $Y = HX + W$.

It is sufficient to prove that H has $\text{rank}(H) \geq \bar{r}T$ since DOF is equal to $\text{rank}(H)$. To do so, we first establish that there exists an assignment of h_i such that $\text{rank}(H) \geq \bar{r}T$.

Let us consider the same $\bar{r}T \times \bar{r}T$ sub-matrix H' by deleting rows and columns in the same way that G' was obtained from G . We have that $\det(H')$ is a multi-variate polynomial in $h_i, i = 1, 2, \dots, N$, if we treat h_i as formal variables. Now this polynomial has integer coefficients and therefore can be treated as a polynomial over any finite field, in particular over the finite field \mathbb{F}_p . Over \mathbb{F}_p , we know that this polynomial is a non-zero polynomial, since the assignment of $h_i = \xi_i$ gives a non-zero value. It follows that this polynomial is nonzero, even when viewed as a polynomial over the integers. Since \mathbb{C} is algebraically closed, we have that any non-zero polynomial must have a assignment of variables in \mathbb{C} that gives non-zero value to the polynomial. Using this assignment for h_i gives us that $\det(H') \neq 0$ and thereby H has $\text{rank}(H) \geq \bar{r}T$.

We have the following lemma:

Lemma 3.5: Consider a multi-variate polynomial f in several variables $h_i, i = 1, 2, \dots, N$. Let h_i be independent random variables in \mathbb{C} generated according to any probability density function. If the polynomial has a non-zero assignment, then the polynomial is non-zero with probability one.

Now using the Lemma above along with the fact that we have an assignment for h_i such that $\text{rank}(H) \geq \bar{r}T$ with probability one. Therefore, for channels with frequency (time) selectivity and coding over multiple frequency (time) slots, we have that the achievable degrees of freedom is equal to $\gamma - \delta$. Since DOF is defined as the supremum over all achievable DOF values, we have that $\text{DOF} = \gamma$ or

$$\text{DOF} = \min_{\omega \in \Omega} \text{rank}(H_\omega)$$

E. Multi-casting over Gaussian Networks

We state the following Theorem without proof:

Theorem 3.6: Given a single-source D -sink multi-cast gaussian wireless network, with independent fading coefficients having an arbitrary density function, the DOF of the network is given by

$$D = \min_{\{j=1,2,\dots,D\}} \min_{\omega \in \Omega_j} \text{rank}(H_\omega) \text{ with probability one} \quad (7)$$

An amplify-and-forward strategy utilizing only linear transformations at the relays is sufficient to achieve this DOF.

IV. CONCLUSION

This paper presented two max-flow min-cut type theorems for computing diversity and DOF of multi-antenna wireless gaussian networks. In addition, a connection was established between DOF of gaussian networks and capacity of deterministic networks [1] for the single-source single-sink and the multi-cast case. Along the way, we proved that the zero error capacity of deterministic networks is the same as the ϵ -error capacity. While the exact evaluation of capacity for the simplest relay networks remains open, approximate high SNR characterizations can be obtained in closed form, even for arbitrary relay networks using simple amplify-and-forward protocols.

REFERENCES

- [1] A. S. Avestimehr, S. N. Diggavi, D. Tse, "Wireless Network Information Flow," *Forty-Fifth Annual Allerton Conference*, Sep 2007
- [2] A. S. Avestimehr, S. N. Diggavi, D. Tse, "A Deterministic Approach to Wireless Relay Networks," *45-th Annual Allerton Conference*, Sep 2007
- [3] A. Host-Madsen and A. Nosratinia, "The multiplexing gain of wireless networks," in *Proc. of ISIT*, 2005.
- [4] V. R. Cadambe, S. A. Jafar, "Interference Alignment and the Degrees of Freedom for the K User Interference Channel," *submitted to IEEE Trans. on Inform. Theory*.
- [5] V. R. Cadambe, S. A. Jafar, S. Shamai, "Interference Alignment on the Deterministic Channel and Application to Fully Connected AWGN Interference Networks," Available Online: <http://arxiv.org/abs/0711.2547>
- [6] M. Godavarti, and A. O. Hero III, "Diversity and Degrees of Freedom in Wireless Communications," in *ICASSP*, May 2002, vol. 3, pp. 2861–2854.
- [7] S. Borade, L. Zheng and R. Gallager, "Amplify and Forward in Wireless Relay Networks: Rate, Diversity and Network Size," *IEEE Trans. on Inform. Theory*, vol 53, no.10, pp 3302-3318, Oct. 2007
- [8] C. E. Shannon, "The zero-error capacity of a noisy channel," *IEEE Trans. Inform. Theory*, vol. 2, no. 3, pp 8–19, Sep 1956.
- [9] L. R. Ford, Jr. and D. R. Fulkerson, "Maximal flow through a network," *Canadian Journal of Mathematics*, Vol.8, pp 399–404, 1956.
- [10] R. Ahlswede, N. Cai, S. Robert Li, and R. W. Yeung, "Network Information Flow," *IEEE Trans. Info. Theory*, Vol.46, No.4, pp.1204-16 July 2000.
- [11] R. Koetter and M. Mardar, "An Algebraic Approach to Network Coding," *IEEE/ACM Trans. Networking*, Vol. 11, No. 5, pp.782-795, Oct 2003.
- [12] B. Nazer and M. Gastpar, "Computation over Multiple-Access Channels," *IEEE Trans. Info. Theory*, Vol.53, No.10, pp.3498-3516, Oct 2007.
- [13] S. Yang and J.-C. Belfiore, "Optimal space-time codes for the MIMO Amplify-and-Forward cooperative channel," *IEEE Transactions on Information Theory*, vol. 53, Issue 2, pp 647-663, Feb. 2007
- [14] S. Yang and J.-C. Belfiore, "Diversity of MIMO Multihop Relay Channels," *submitted to IEEE Trans. on Inform. Theory*, Available Online: <http://arxiv.org/abs/0708.0386>
- [15] T. M. Cover, J. A. Thomas, *Elements of Information Theory*, 2nd Edition, John Wiley and Sons, 2006.
- [16] L. Zheng and D. Tse, "Diversity and Multiplexing: A Fundamental Tradeoff in Multiple-Antenna Channels," *IEEE Trans. Inform. Theory*, vol. 49, no. 5, pp. 1073–1096, May 2003.